

# Lowest-Lying Scalar Mesons and a Possible Probe of Their Quark Substructure <sup>1</sup>

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**Abstract.** In this talk, an overview of the status of the light scalar mesons in the context of the non linear chiral Lagrangian of references [1–3] is presented. The evidence for the existence of a scalar nonet below 1 GeV is reviewed, and it is shown that by introducing a scalar nonet an indirect way of probing the quark substructure of these scalars through the scalar mixing angle can be obtained. It is then reviewed that consistency of this non-linear chiral Lagrangian framework with the experimental data on  $\pi\pi$  and  $\pi K$  scattering, as well as the decay  $\eta' \rightarrow \eta\pi\pi$ , results in a range for the mixing angle which indicates that the quark substructure of these light scalars are closer to a four quark picture.

## I INTRODUCTION

Lowest-lying scalar mesons (scalar mesons below 1 GeV) are of fundamental importance in understanding the theory and phenomenology of low energy QCD. However, the properties of these scalars, in particular their quark substructure, are not quite understood. As a result they are at the focus of many theoretical and experimental investigations.

From the experimental point of view, there are at least four well established light scalars – the isosinglet  $f_0(980)$  and the isotriplet  $a_0(980)$ . There are also five other candidates – the isosinglet  $\sigma(560)$ , and two isodoublets  $\kappa(900)$  and  $\bar{\kappa}(900)$ , which are not quite established experimentally. In 1998 edition of Particle Data Group [4], the  $\sigma(560)$  is listed as  $f_0(400 - 1200)$  with a very uncertain properties; a mass between 400 to 1200 MeV and a very broad decay width between 600 to 1000 MeV. In a recent experimental study of  $\tau$  lepton decay by CLEO collaboration [5], a significant contribution due to the  $\sigma$  is pointed out and it is reported that inclusion of a  $\sigma$  with  $m_\sigma = 555$  MeV and  $\Gamma_\sigma = 540$  MeV significantly improves the fits. The situation of  $\kappa(900)$  is not clear experimentally. Therefore, altogether there are 9 possible candidates for lowest lying scalar mesons.

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The situation of these scalars is also not clear from the theoretical point of view; there are model dependent calculations, and as a result, different conclusions. However, within any theoretical framework two basic questions should be addressed:

1. Is there a clear evidence for the existence of a  $\sigma(560)$  and a  $\kappa(900)$ ?
2. Can we describe the properties of these mesons like their masses, decay widths, interactions, and in particular their quark substructure?

In this talk we first review the general nonlinear chiral Lagrangian framework of refs. [1–3], and discuss how within this framework a  $\sigma(560)$  was observed in [6,7], and a  $\kappa(900)$  was observed in [1]. We then review how this nonlinear chiral Lagrangian can be rewritten in terms of a scalar nonet by introducing a few new free parameters. We show how these parameters as well as the acceptable range of the scalar mixing angle can be fixed by considering the  $\pi\pi$  and  $\pi K$  scattering, and the  $\eta' \rightarrow \eta\pi\pi$  decay. Based on the acceptable range of the scalar mixing angle we discuss that the quark substructure of the light scalar mesons is closer to a four quark picture. We conclude by summarizing the results.

## II OUR THEORETICAL FRAMEWORK AND EVIDENCE FOR THE $\sigma(560)$ AND $\kappa(900)$

We work within the effective non-linear chiral Lagrangian framework. The pseudoscalar part of the Lagrangian is [1]

$$\mathcal{L}_\phi = -\frac{F_\pi^2}{8} \text{Tr} \left( \partial_\mu U \partial_\mu U^\dagger \right) + \text{Tr} \left[ \mathcal{B} (U + U^\dagger) \right] \quad (1)$$

with  $F_\pi = 131$  MeV,  $\mathcal{B} = m_\pi^2 F_\pi^2 / 8 \text{diag}(1, 1, 2m_K^2/m_\pi^2 - 1)$  is the dominant symmetry breaking term, and  $U = e^{2i\frac{\phi}{F_\pi}} = \xi^2$  where  $\phi$  is the pseudoscalar nonet

$$\phi_a^b = \begin{bmatrix} \frac{\eta_{NS} + \pi_0^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_{NS} - \pi_0^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{bmatrix}. \quad (2)$$

$U$  transforms linearly under chiral transformation [ $U \rightarrow U_L U U_R^\dagger$  with  $U_{L,R} \in U(3)_{L,R}$ ], whereas  $\xi$  transforms nonlinearly [ $\xi \rightarrow U_L \xi K^\dagger(\phi, U_L, U_R) = K(\phi, U_L, U_R) \xi U_R^\dagger$ ].

The vectors can be introduced in this framework in terms of the vector nonet  $\rho$  with a Lagrangian that has the same form as that of usual gauge fields

$$\mathcal{L}_\rho = -\frac{1}{2} m_v^2 \text{Tr} \left[ \left( \rho_\mu - \frac{v_\mu}{\tilde{g}} \right)^2 \right] - \frac{1}{4} \text{Tr} [F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)] \quad (3)$$

with  $F_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu]$ . In (3),  $p_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi)$  and  $v_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi)$ , and have simple transformation properties under chiral transformation.

We introduce scalars into this picture in two stages. First in order to see whether there is an indication of  $\sigma(560)$  and  $\kappa(900)$  within our framework, we introduce scalars in a phenomenological way. We consider a general isospin invariant form [1]

$$\begin{aligned}\mathcal{L}_s = & -\frac{\gamma_{\sigma\pi\pi}}{\sqrt{2}}\sigma\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} - \frac{\gamma_{\sigma K\bar{K}}}{\sqrt{2}}\sigma\left(\partial_\mu K^+\partial_\mu K^- + \dots\right) - \frac{\gamma_{f_0\pi\pi}}{\sqrt{2}}f_0\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} \\ & - \frac{\gamma_{f_0 K\bar{K}}}{\sqrt{2}}f_0\left(\partial_\mu K^+\partial_\mu K^- + \dots\right) - \gamma_{\kappa K\pi}\left(\kappa^0\partial_\mu K^-\partial_\mu\pi^+ + \dots\right)\end{aligned}\quad (4)$$

and take the coupling constants as independent parameters – we either take the couplings as fitting parameters or input them from experimental measurements.

Now in order to see whether this framework sees a  $\sigma$  and/or a  $\kappa$ , we can consider computing processes to which these mesons could significantly contribute, and then compare our prediction to the experimental data and search for signs of these scalars. In principle  $\sigma(560)$  and  $\kappa(900)$  can be probed in  $\pi\pi$  and  $\pi K$  scattering, respectively. In fact, their contributions could be substantial as they appear as poles in the scattering amplitudes. To see the effect of  $\kappa$ , using Lagrangian (4), appropriate  $\pi K$  scattering amplitudes were computed in [1] and the result were matched to the experimental data. The scattering amplitudes were computed by only taking tree level Feynman diagrams into account – this is motivated by  $1/N_c$  expansion. It was shown in [1] that a  $\kappa$  with a mass around 900 MeV and a decay width around 320 MeV is needed in order to describe the experimental data on the  $\pi K$  scattering. This technique was first developed in [6,7], in which it was shown that in order to agree with the  $\pi\pi$  experimental data there is a need for a  $\sigma$  meson with a mass around 550 MeV and a decay width around 370 MeV.

Therefore within our theoretical framework there is a clear signal for both the  $\sigma(560)$  and the  $\kappa(900)$ . This motivates us, in our second stage of investigation, to combine these scalars together with the  $f_0(980)$  and the  $a_0(980)$  into a light scalar nonet, in terms of which we rewrite our Lagrangian in the next section.

### III A POSSIBLE SCALAR NONET BELOW 1 GEV

We now construct a light scalar nonet out of  $\sigma(560)$ ,  $\kappa(900)$ ,  $f_0(980)$  and  $a_0(980)$  in the form

$$N = \begin{bmatrix} N_1^1 & a_0^+ & \kappa^+ \\ a_0^- & N_2^2 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & N_3^3 \end{bmatrix}. \quad (5)$$

In general,  $\sigma(560)$  and  $f_0(980)$  are a mixture of  $(N_1^1 + N_2^2)/\sqrt{2}$  and  $N_3^3$ . We represent this mixing in terms of a scalar mixing angle  $\theta_s$  as

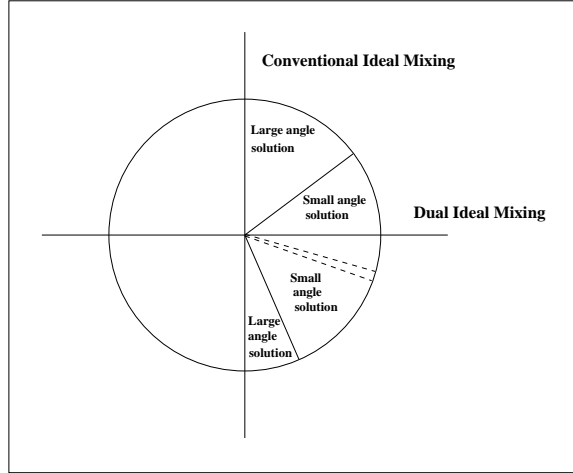
$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} N_3^3 \\ \frac{N_1^1 + N_2^2}{\sqrt{2}} \end{pmatrix}. \quad (6)$$

In this parametrization,  $\theta_s = \pm\pi/2$  corresponds to the conventional ideal mixing when a pure  $q\bar{q}$  assignment is used for  $N$ . Another interesting limit is  $\theta_s = 0$  which corresponds to the dual ideal mixing when a pure four-quark assignment is used to describe  $N$  [8].

We can now rewrite the scalar sector of our Lagrangian in terms of the nonet (5)

$$\begin{aligned}\mathcal{L}_{mass} &= -a\text{Tr}(NN) - b\text{Tr}(NN\mathcal{M}) - c\text{Tr}(N)\text{Tr}(N) - d\text{Tr}(N)\text{Tr}(N\mathcal{M}) \\ \mathcal{L}_{N\phi\phi} &= A\epsilon^{abc}\epsilon_{def}N_a^d\partial_\mu\phi_b^e\partial_\mu\phi_c^f + B\text{Tr}(N)\text{Tr}(\partial_\mu\phi\partial_\mu\phi) \\ &\quad + C\text{Tr}(N\partial_\mu\phi)\text{Tr}(\partial_\mu\phi) + D\text{Tr}(N)\text{Tr}(\partial_\mu\phi)\text{Tr}(\partial_\mu\phi)\end{aligned}\quad (7)$$

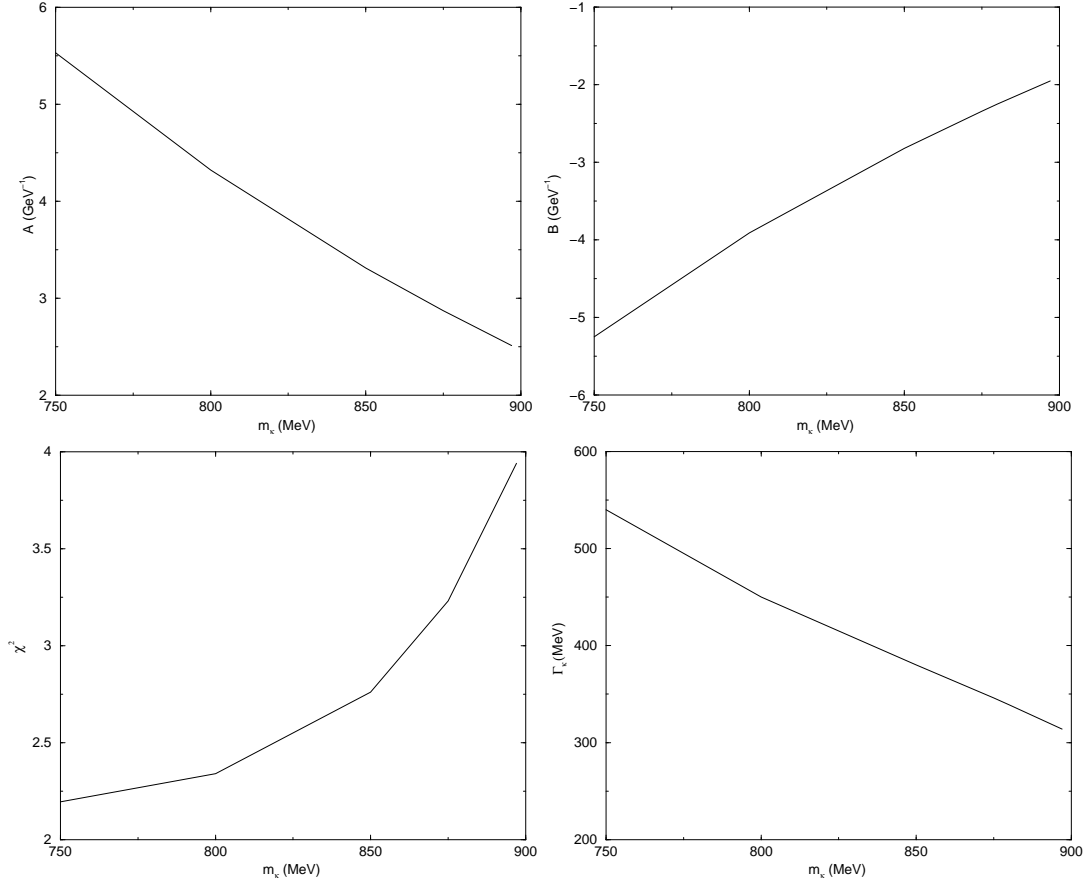
with  $\mathcal{M} = \text{diag}(1, 1, x)$  is the spurion matrix with  $x$  the ratio of strange to non-strange quark masses. The mass part of the Lagrangian is given in terms of new free parameters  $a, b, c, d$ , and  $\theta_s$  which can be determined by inputting the scalar masses  $m_\sigma, m_{f_0}, m_\kappa$  and  $m_{a_0}$ . We find that our model restricts  $m_\kappa$  in the range 685 to 980 MeV, and also for any input of scalar masses there are two solutions for  $\theta_s$ . Thus, in our framework there are two acceptable ranges for the scalar mixing angle which are shown in Fig. 1. These are the *large angle solution*:  $36^\circ \leq \theta_s \leq 90^\circ$  and  $-90^\circ \leq \theta_s \leq -71^\circ$ , and the *small angle solution*:  $-71^\circ \leq \theta_s \leq 36^\circ$ . As  $m_\kappa$  varies from its minimum value to its maximum, these regions are entirely swept through. There are also new free parameters  $A, B, C$ , and  $D$  in the scalar-pseudoscalar-pseudoscalar interaction part of Lagrangian which can be determined



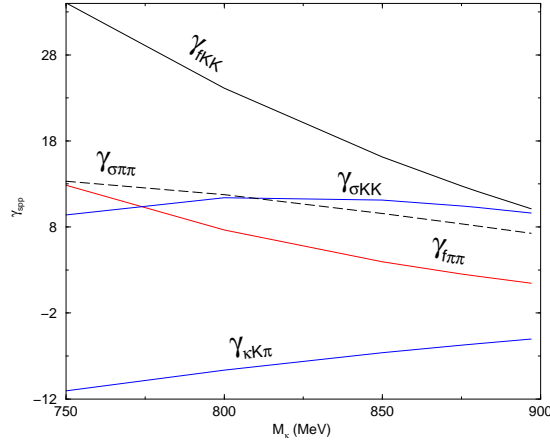
**FIGURE 1.** Two regions for the scalar mixing angle for the acceptable range of  $685 \text{ MeV} \leq m_\kappa \leq 980 \text{ MeV}$ . As  $m_\kappa$  varies from its minimum value to its maximum value,  $\theta_s$  in the small angle region varies from  $36^\circ$  to  $-71^\circ$ , and in the large angle region  $\theta_s$  varies from  $36^\circ$  to  $90^\circ$ , then to  $-90^\circ$  and finally to  $-71^\circ$ . The region bounded between dashed lines ( $-20^\circ \leq m_\kappa \leq -15^\circ$ ) corresponds to  $875 \text{ MeV} \leq m_\kappa \leq 897 \text{ MeV}$ , and is consistent with experimental data on  $\pi\pi$  and  $\pi K$  scattering, as well as on  $\Gamma[f_0(980) \rightarrow \pi\pi]$ .

by appropriately matching our theoretical prediction to the experimental data. A consequence of introducing the nonet (5) is that the scalar couplings are now related to each other by the underlying chiral symmetry, i.e.  $\gamma_{spp} = \gamma_{spp}(A, B, C, D, \theta_s, \theta_p)$ , where  $\theta_p$  is the pseudoscalar mixing angle for which we choose the value  $37^\circ$ .

We numerically search through both ranges of  $\theta_s$  and fit our prediction for  $\pi K$  scattering amplitude to the experimental data. This determines  $A$  and  $B$  in the interaction Lagrangian. We find that the  $\chi^2$  of fit improves as we lower  $m_\kappa$ . This is shown in Fig. 2, together with the  $m_\kappa$  dependence of the parameters  $A$  and  $B$  in the interaction Lagrangian, and the total decay width of  $\kappa$ . Although the  $\chi^2$  fit improves for lower values of  $m_\kappa$ , other experimental data further restricts the acceptable range of  $m_\kappa$ . We take into account limits from  $\pi\pi$  scattering amplitude [6,7] on the strong interaction couplings  $\gamma_{spp}$ , as well as the decay width  $\Gamma[f_0(980) \rightarrow \pi\pi]$ . The  $m_\kappa$  dependence of the resulting couplings are shown in



**FIGURE 2.**  $m_\kappa$  dependence of the fitting parameters  $A$ ,  $B$  and  $\Gamma_\kappa$ , in a fit of the theoretical prediction of the  $\pi K$  scattering amplitude to the experimental data, together with the  $\chi^2$  of the fit.

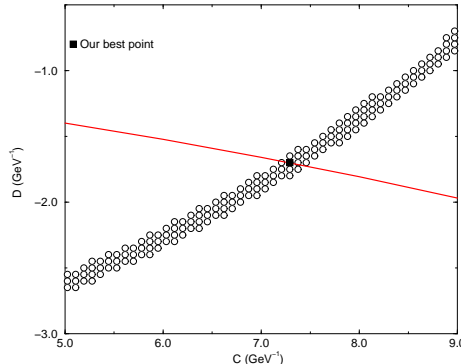


**FIGURE 3.**  $m_\kappa$  dependence of the hadronic couplings found in a fit of the theoretical prediction of the  $\pi K$  scattering amplitude to the experimental data.

Fig. 3. We find that the small angle solution is favored as it contains a small region (corresponding to  $875 \text{ MeV} \leq m_\kappa \leq 897 \text{ MeV}$ ) that is consistent with these experimental constraints. This region, which is bounded by dashed lines in Fig. 1, is obviously close to  $\theta_s = 0$ . This is how our model indirectly probes the quark substructure of this scalar nonet – the fact that  $\theta_s$  is small means that the scalar mixing in our model is closer to the dual ideal mixing and therefore a four-quark scenario is favored for this nonet.

#### IV $\eta' \rightarrow \eta\pi\pi$ DECAY

In the previous section we rewrote the scalar part of the Lagrangian in terms of a scalar nonet  $N$ . We evaluated all free parameters in the Lagrangian except  $C$  and  $D$  in the interaction piece in (7). These two parameters were probed in ref. [3] by matching the prediction of our model for the partial decay width of  $\eta' \rightarrow \eta\pi\pi$ , and for the energy dependence of the normalized magnitude of the decay matrix element, to the experimental data. The same values of  $A$ ,  $B$  and  $\theta_s$  that were found in ref. [2] were used in this decay analysis. The  $CD$  parameter space was scanned numerically and was searched for the physical regions that describe both experimental measurements of this decay. The result is shown in Fig. 4. The gray region is consistent with the partial decay width of this decay, and the solid line is consistent with the energy dependence of the normalized magnitude of the decay matrix element. Their intersection in the  $CD$  plane ( $C \approx 7.3 \text{ GeV}^{-1}$  and  $D \approx -1.7 \text{ GeV}^{-1}$ ) exists and is *unique*. This means that there is a unique choice of free parameters of Lagrangian (7) that, in addition to  $\pi\pi$  and  $\pi K$  scattering amplitudes, describes the  $\eta' \rightarrow \eta\pi\pi$  decay. The energy dependence of this decay is plotted in Fig. 5.

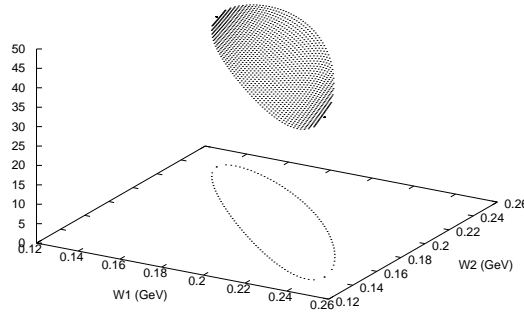


**FIGURE 4.** Regions in  $CD$  plane consistent with two different experimental measurements on  $\eta'$  decay. Regions consistent with  $\Gamma^{exp.}[\eta' \rightarrow \eta\pi\pi]$  are represented by circles. The solid line represents points consistent with the energy dependence of the normalized magnitude of the decay matrix element.

As a by-product we compute, with the same extracted  $C$  and  $D$ , the partial decay width of  $a_0(980) \rightarrow \pi\eta$  to be approximately 65 MeV. This, together with the  $\Gamma[a_0(980) \rightarrow K\bar{K}] \approx 5 \text{ MeV}$  found in [2], provide an estimate of the total decay width of  $a_0(980)$  around 70 MeV. This is in a very close agreement with a recent experimental analysis of  $a_0(980)$  in ref. [9].

## V SUMMARY AND DISCUSSION

In this talk we reviewed the light scalar mesons in the non-linear chiral Lagrangian framework of references [1–3]. We saw that in this approach there is a need for the  $\sigma(560)$  and the  $\kappa(900)$  in order to be able to describe the experimental data on the  $\pi\pi$  and  $\pi K$  scattering. We then constructed a light scalar nonet below 1 GeV consisting of  $\sigma(560)$ ,  $\kappa(900)$ ,  $f_0(980)$ , and  $a_0(980)$ , and rewrote the Lagrangian in terms of this nonet by introducing eight new free parameters. We showed that with these parameters we can describe many experimental facts including the scalar mass spectrum, their interactions with pseudoscalars in  $\pi\pi$  and  $\pi K$  scattering, the  $\eta'$  decay, and the decay width of  $f_0(980)$ . We could predict the total decay width of  $a_0(980) \approx 70$  MeV in a very close agreement with a recent experiment [9]. We discussed that although the chiral Lagrangian model presented here is entirely formulated in terms of the meson fields and in principle does not know anything about the underlying quark substructure, in practice, the knowledge of the mixing angle indirectly probes the quark substructure of these scalars. We saw, through a careful numerical analysis, that the acceptable range of the mixing angle is such that it suggests the quark substructure of these scalars is closer to



**FIGURE 5.** Energy dependence of the magnitude of the  $\eta' \rightarrow \eta\pi\pi$  decay matrix element.  $w_1$  and  $w_2$  are the total energy of the final state pions, and are bounded within the ellipse-like region in the  $w_1 w_2$  plane.

a four quark picture. This is in agreement with a recent theoretical investigation [10].

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